

Geometric Suppression of Single-Particle Energy Spacings in Quantum Antidots

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Abstract

Quantum Antidot (AD) structures have remarkable properties in the integer quantum Hall regime, exhibiting Coulomb-blockade charging and the Kondo effect despite their open geometry. In some regimes a simple single-particle (SP) model suffices to describe experimental observations while in others interaction effects are clearly important, although exactly how and why interactions emerge is unclear. We present a combination of experimental data and the results of new calculations concerning SP orbital states which show how the observed suppression of the energy spacing between states can be explained through a full consideration of the AD potential, without requiring any effects due to electron interactions such as the formation of compressible regions composed of multiple states, which may occur at higher magnetic fields. A full understanding of the regimes in which these effects occur is important for the design of devices to coherently manipulate electrons in edge states using AD resonances.

Key words: Antidot, Edge-states, Aharonov-Bohm

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Several exciting developments in the field of mesoscopic physics have recently emerged from the study of coherent electronic devices [1]. By utilising coherent edge states in quantum Hall systems, devices can be constructed which coherently manipulate electron spin, charge, and phase, offering important prospects for the study of quantum information in the solid state. Quantum antidots in particular offer spin-selective coherent control of these states, making them potentially important in future applications [2]. An antidot (AD) is simply a potential hill in a two-dimensional elec-

tron system, which in a perpendicular magnetic field ($\gtrsim 0.3$ T) gains a set of quasibound states quantised by the Aharonov-Bohm (AB) effect. When placed at the centre of a constriction, these states couple to extended edge channels, and their structure may then be investigated through the scattering processes which determine the conductance. In the simplest model, these noninteracting single-particle (SP) states adjust with changing magnetic field in order to maintain their enclosed flux, producing a set of magnetoconductance oscillations as they pass through the Fermi energy (E_F) in the leads. Many details of AD behaviour may be understood within this SP picture, particularly at low fields

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($\lesssim 3$ T) [3]. Other effects, such as Coulomb Blockade [4] and the Kondo Effect [5] require a self-consistent model including electron interactions.¹ There has been some debate over the precise nature of these effects [7,8,9], particularly concerning the presence or absence of compressible regions (CRs) composed of multiple partially-occupied SP states within a few $k_B T$ of E_F . Certainly, in the limit of large AD size and high field, CRs are expected to form in order to minimise the Coulomb energy associated with abrupt changes in electron density [10], but these may be suppressed in other regimes by the AB quantisation and exchange effects. Recent numerical calculations accounting for both Coulomb and spin-dependent interactions confirm this expectation [11]; for an AD of radius 200 nm in the integer quantum Hall regime with filling factor $\nu = 2$, CRs are significantly quenched for fields below ≈ 4 T, and then a CR only forms for the outer spin state.

We present a series of measurements designed to explore the evolution of SP states with magnetic field. Our device² consists of an AD gate (200 nm in diameter) centered in a 1- μm channel, contacted by a metal gate isolated from the split gate by an insulating layer of crosslinked polymer, allowing us to control the voltage on the AD independently. As a function of B and source-drain bias, the conductance forms a complicated pattern of resonances not unlike that observed in quantum dots. From these data, we can extract the various transition energies of the system — the Coulomb energy E_C , the Zeeman spin-splitting E_Z , and the SP energy ΔE_{SP} . As was noted in [12], at higher fields we observe a clear suppression of ΔE_{SP} below the expected $1/B$ dependence for circular AD states, and it was then suggested that this could reflect the formation of CRs. We have investigated this further, however, and find that this suppression occurs systematically on the high- B end of the $\nu = 2$ plateau, coinciding with the transition of AB resonances from transmission peaks to reflection valleys. We have also seen this effect at relatively low fields $\lesssim 2$ T (see Fig. 1) where CRs are less likely to form. We therefore propose an alternative explanation for this effect, namely that the presence of the split-gate spreads out the AB states in the constrictions, leading to a suppression of ΔE_{SP} for the lowest SP states in a Landau level.

¹ For a recent review see [6].

² Complete details may be found in [12].

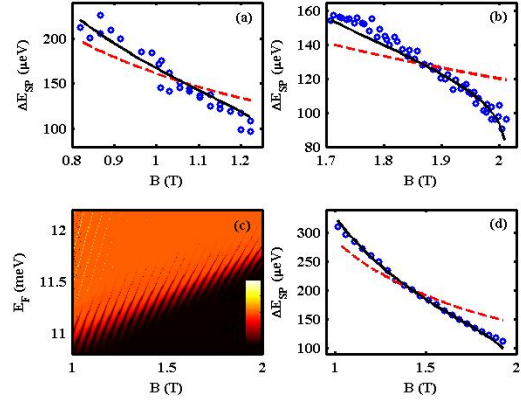


Fig. 1. Top panels: Single-particle energy ΔE_{SP} (circles) extracted from DC-bias measurements in different ranges of magnetic field (with different gate voltages). A $1/B$ fit (dashed curve) fails to match the data while our model (solid curve) predicts the reduction of ΔE_{SP} at higher fields. Bottom panels: Conductance (c) in units of $2e^2/h$ as a function of B and E_F , calculated from the full (non-interacting) Green’s function computed using an iterative procedure [13], and the corresponding energy spacing (d), calculated at $E_F = 11.7$ mV. As in the experimental data, our model (solid curve) accounts for the discrepancy from the $1/B$ dependence (dashed curve).

It is straightforward to show that the asymmetry introduced by the constrictions leads to a suppression of ΔE_{SP} . The spatial separation of SP states is determined by the additional flux h/e enclosed by each successive orbit, and the “bulging” of states lower in the constrictions accounts for an increased fraction of the area. Since the states are contours of the AD potential and the slope far away from the constrictions is nearly constant, the energy spacing between these states is reduced (see Fig. 2). For an AD potential $U(r, \theta)$ which varies on a scale much larger than the separation between states, we can calculate

$$\Delta E_{\text{SP}} = -\frac{h}{eB} \left[\int_0^{2\pi} \left(\frac{dU}{dr} \right)_{(C, \theta)}^{-1} \mathcal{C}(\theta) d\theta \right], \quad (1)$$

where $\mathcal{C}(\theta)$ is the contour at the Fermi energy defined by $U_{\text{eff}}(\mathcal{C}, \theta) = E_F$, using the effective potential $U(r, \theta) + E_{\text{cyc}} + E_Z$, where E_{cyc} and E_Z are the cyclotron and Zeeman energies for states in the constrictions. For a circularly symmetric potential this gives the $1/B$ dependence mentioned above,³ but it is clear

³ The B -dependence of $\mathcal{C}(\theta)$ is quite weak.

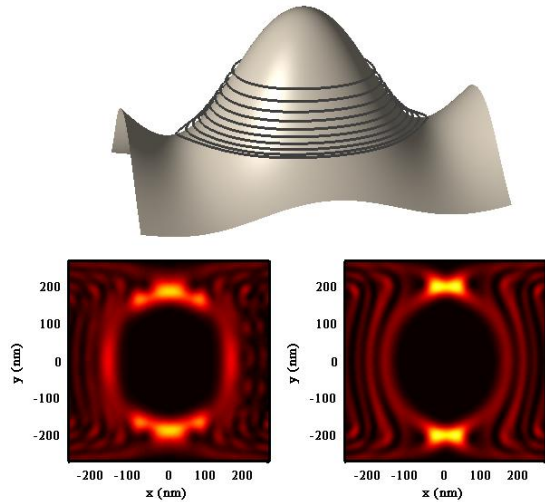


Fig. 2. Top: Potential created by an AD in a constriction (computed as in [14]), with SP contours calculated according to Eq. (1). Note the “bulging” of the contours into the constrictions, which results in a much-reduced energy spacing. Bottom: Local density of states calculated from the noninteracting Green’s function for the AD shown above. Note that the transmission resonance (left panel) at low field (≈ 1 T) is more circularly symmetric than the reflection resonance (right panel) at higher field (≈ 1.5 T).

that a reduction in the slope of the potential over any region of the contour results in a suppressed ΔE_{SP} . It is worth noting that the magnetic flux does not solely determine the location of AD states, since the nonuniform AD potential contributes an additional phase factor to the wavefunctions beyond the magnetic AB phase,⁴ but for a potential that varies sufficiently slowly, the dominant contribution to the difference between adjacent states arises from the change in magnetic flux, and so a model considering only differences in area is appropriate.

Using the bare electrostatic potential [14] resulting from the gates on our device, we can use Eqn. 1 to calculate ΔE_{SP} as a function of B to compare with the data. In Fig. 1 (a and b) we show the results of this calculation for two data sets with different experimental parameters, and a comparative best-fit curve $\propto 1/B$. The model itself has no free parameters; the potential

is completely determined by the arrangement of gates, the measured AB period (related to the AD area by $A\Delta B = h/e$), and ΔE_{SP} at low field, and we can estimate E_{F} from the field at which the $\nu = 2$ state is depopulated in the channel. We have included no effects of tunnelling between SP states and the leads in this calculation, which results in an artificial drop to zero as the saddle point reaches E_{F} and closed orbits no longer exist. For additional comparison with this essentially classical model, we have calculated the full (noninteracting) Green’s function for an AD + split-gate geometry using an iterative procedure [13]. Fig. 2 shows the calculated local density of states for an AD device at a reflection resonance, and Fig. 1c shows the calculated conductance as a function of B and E_{F} . From the conductance we can calculate ΔE_{SP} , as shown in Fig. 1d, and we find a nearly identical suppression at higher fields which is well matched by our geometric model. Since this calculation includes no electron interaction effects, we can be sure that CRs are not required to produce this behaviour.

We therefore conclude that the observed suppression of ΔE_{SP} is a simple result of the potential profile in our experimental geometry, rather than a signature of a reorganisation of states into a CR. Although we know that interaction effects become essential for an understanding of AD resonances at high fields, this study demonstrates the capacity of the SP model to explain relatively complicated features of the excitation spectrum of ADs in the low-field regime. An understanding of this structure is critical in the design of AD-based applications which seek to utilise processes in a specific regime. By choosing AD sizes and fields appropriately, it may be possible to construct devices which use ADs in different regimes (even on a single chip) to manipulate spin or charge for different purposes. These avenues of research remain largely unexplored, and may hold many exciting advances in the study of quantum-coherent electronics.

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⁴ This also means that AD states do not enclose integer multiples of the flux quantum h/e as commonly assumed, but rather are pushed outwards by the AD potential.

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